Problem 1. *Suppose that* $\mathfrak{A}$ *and* $\mathfrak{B}$ *are two models in a language* $\mathcal{L}$*, such that there is a bijection* $h : |\mathfrak{A}| \to |\mathfrak{B}|$*, such that:*

- *for each constant symbol* c *in* $\mathcal{L}$*,* $h(c^{\mathfrak{A}}) = c^{\mathfrak{B}}$*,*
- *for each* n*-place predicate* P *in* $\mathcal{L}$*,*

$$\langle a_1, ..., a_n \rangle \in P^{\mathfrak{A}} \textit{ iff } \langle h(a_1), ..., h(a_n) \rangle \in P^{\mathfrak{B}},$$

- *for every* n*-place function symbol* f *in* $\mathcal{L}$*,*

$$h(f^{\mathfrak{A}}(a_1, ..., a_n)) = f^{\mathfrak{B}}(h(a_1), ..., h(a_n)).$$

Prove that for every formula ϕ *with free variables* $x_1, ..., x_n$*, and for every* $a_1, ..., a_n$ *in* $|\mathfrak{A}|$*, we have* $\mathfrak{A} \models \phi[a_1, ..., a_n]$ *iff* $\mathfrak{B} \models \phi[h(a_1), ..., h(a_n)]$*.*

Remark 1. Such a function is called an isomorphism, and if it exists, we say that the models are isomorphic, denoted by $\mathfrak{A} \cong \mathfrak{B}$.